Int. J. Multiphase Flow Vol. 13, No. 2, pp. 233–246, 1987
Printed in Great Britain.

0301-9322/87 $3.00 + 0.00

FLUID FLOW STRUCTURE IN A DILUTE TURBULENT TWO-PHASE SUSPENSION FLOW IN A VERTICAL PIPE

S. L. LEE† and T. BÖRNER
Lehrstuhl für Strömungsmechanik, Universität Erlangen-Nürnberg, Egerlandstraße 13, 8520 Erlangen, B.R.D.

(*Received* 17 *September* 1985; *in revised form* 7 *July* 1986)

Abstract—The dynamics of a turbulent two-phase suspension flow plays a fundamental role in the study of a large number of technical problems of great importance. In order to achieve a more comprehensive understanding of the particle motion in such a flow, a few of the key flow properties of the fluid in a suspension mixture are required. These flow properties are determined from an analysis of the results of some recent local measurements of such a flow system using non-intrusive laser–Doppler anemometry and the required correlations are produced in terms of the physical and dynamic characteristics.

INTRODUCTION

The deposition of solid particles or droplets from a turbulent particle–suspension flow at the channel walls is a problem of fundamental importance in a number of technical areas. Although a large number of articles on this subject have appeared in the literature, reliable results from carefully planned experiments are scarce and mostly relate only to the amount of deposition at the wall without elucidating the mechanisms in the flow that are responsible for the transport of the particles towards the wall; e.g. the measurements of the deposition of relatively large droplets from a gas by Alexander & Coldren (1951) and Cousins & Hewitt (1968), and of small solid particles from air by Friedlander & Johnstone (1957).

Most theoretical treatments of the subject adopt the point of view of a conventional three-layer flow structure in the vicinity of the wall—the viscous sublayer, the buffer zone and the turbulent core—from studies of single-phase, fully-developed turbulent flow, e.g. Friedlander & Johnstone (1957), Lin *et al.* (1953), Davis (1966), Beal (1968) and Hutchinson *et al.* (1971). In the turbulent core and the buffer zone, particles are assumed to be laterally transported by turbulent diffusion in the same way as scalar quantities, such as heat or concentration of species, are assumed to be transported in a turbulent stream. Particles reaching the edge of the viscous sublayer as a result of this transport are assumed to coast towards a wall across the sublayer to form deposition. A common feature of these treatments is their possession of an adjustable empirical factor, which is necessary to achieve a reasonable comparison between the theoretically predicted and experimentally determined amounts of deposition for a particular flow system. Unfortunately, this empirical factor is in no case a universal constant for all flow systems. This obvious inconsistency leads one to question the correctness of the very physics assumed in these theoretical treatments, particularly the assumed particle transport by turbulent diffusion, irrespective of particle size, and the ratio of its density to that of the fluid and the assumed absence of influence on the flow structure of the fluid due to the presence of particles in the suspension.

In a study of a behaviour particle in turbulent flow, Rouhiainen & Stachiweicz (1970) used the concept of the frequency response of the particle in an oscillating field, first developed by Hjelmfelt & Mokros (1966) based on the work of Tchen (1949). In an attempt to apply the concept of the frequency response of a particle to the practical problem of particle deposition, Lee & Durst (1979, 1982) introduced the simplifying model of particle response characterized by a cutoff frequency, below which the particle responds fully to the fluid oscillation and above which the particle is totally

†On sabbatical leave from Department of Mechanical Engineering, State University of New York at Stony Brook, Stony Brook, NY 11794, U.S.A.

insensitive to the fluid oscillation. Therefore, for fluid oscillation frequencies below the cutoff frequency, the particle motion is determined by turbulent diffusion, whereas for fluid oscillation frequencies above the cutoff frequency, the motion of the particle is controlled by the mean, or quasi-laminar, motion of the fluid. Matching of this cutoff frequency and the most energetic fluctuating frequency from the turbulent motion of the fluid of a single-phase flow produces the cutoff radius, with consideration for the particle size, the physical properties of the particle and the fluid and the flow properties. Within the cutoff radius lies the turbulent diffusion core and outside the annular quasi-laminar region. However, this treatment still suffers from the shortcoming of the assumed absence of influence on the flow structure of the fluid due to the presence of particles in the suspension.

It is clear that, in order to achieve a better understanding of the dynamics of a turbulent two-phase suspension flow, a determination of the flow structure of the fluid in the presence of the suspended particles in such a flow is required. Due to its extreme complexity, such a problem will doubtlessly remain beyond the reach of rigorous theoretical attempts for a long time to come. Most of the previous experimental work on flow with particles concerns either the flow around a single particle in an otherwise quiescent body of ambient fluid or the gross integral flow properties of a two-phase suspension flow. Only recently have new non-intrusive experimental techniques, such as laser–Doppler anemometry, emerged which can take local measurements, simultaneously, of the velocities of the two separate phases together with the size and concentration of the particulate phase in a dilute turbulent two-phase suspension flow.

RECENT EXPERIMENTS ON LOCAL MEASUREMENTS (LDA)

Fairly recently there appeared in the literature two reports on local measurements of flow properties of an upward solid particle–air two-phase turbulent suspension flow in a vertical pipe using non-intrusive laser–Doppler anemometry.

Lee & Durst (1982) described experiments with the flow of air carrying in suspension single-sized glass spheres of 100, 200, 400 and 800 μm in a vertical pipe of 41.8 mm dia. The anemometer was provided with amplitude discrimination and frequency filter banks which together were used to differentiate between fluid and particle velocities. Signals were registered on a transient recorder and transferred to a computer for processing. Profiles of time-mean longitudinal velocities were measured both for the air and the particles for all particle sizes. Profiles of longitudinal velocity fluctuations were also measured both for the air and the particles for a particle size of 800 μm.

Tsuji *et al.* (1984) described experiments with the flow of air carrying in suspension single-sized plastic spheres of 200 μm, 500 μm, 1 mm and 3 mm in a vertical pipe of 30.5 mm dia. A frequency tracker converted the frequency of a Doppler signal into a voltage output proportional to the velocity. The Doppler signal was obtained not only from the small tracers but also from the spherical test particles. The separation of the signals of the test particles from those of the tracers was made possible by a specially designed signal-discrimination device. The principle of the signal discriminator is that the burst signals with sufficiently large pedestal components comes from the large particles, while those with small pedestal but large Doppler components come from the small tracers that follow the fluid motion. Profiles of time-mean longitudinal velocities were measured both for the air and the particles for the three particle sizes of 200 μm, 500 μm and 3 mm, each with three different particle loadings. Profiles of the longitudinal velocity fluctuations of the air and their frequency power spectrum were also measured for all particle sizes, each with three different particle loadings.

LONGITUDINAL DYNAMIC INTERACTION BETWEEN PHASES (DRAG)

The drag between the phases plays an important role in the study of a turbulent two-phase suspension flow. The conventional practice has been to use results of measurements based on a single particle in a laminar stream without the presence of other particles. An apparent contribution to the difference between the drag coefficient for a particle in a two-phase suspension and that for a single particle in a uniform flow is the presence of other particles in the flow which occupy a finite amount of volume in the mixture. Ishii & Zuber (1979) proposed that in the viscous regime the drag coefficient C_D for a turbulent two-phase suspension flow has exactly the same functional form in terms of a modified particle Reynolds number as the drag coefficient for a single particle in a uniform flow in terms of the particle Reynolds number, based on the molecular viscosity of the fluid μ_f, the standard drag curve. The modified Reynolds number is based on a modified viscosity μ_{fm} of the fluid, due the particle volumetric concentration α associated with the presence of other particles in the mixture [as proposed by Ishii (1977)] which is for solid particles:

$$\frac{\mu_{fm}}{\mu_f} = \left(1 - \frac{\alpha}{0.62}\right)^{-1.55}.$$

Now, we apply this proposed drag coefficient to the results of local measurements of the upward turbulent flow of a solid particle–air two-phase suspension in a vertical pipe using non-intrusive laser–Doppler anemometry by Lee & Durst (1982) and Tsuji *et al.* (1984). The particle volumetric concentration α in these experiments varies between 0.6×10^{-3} and 8.0×10^{-3} and the ratio μ_{fm}/μ_f computed from the above equation will vary between 1.001 and 1.02. The computed modified particle Reynolds number will differ from the particle Reynolds number by a maximum of only 2%. Consequently, the discrepancy between the experimental results and predicted values using the proposal on the drag coefficient C_D is still significant, mostly by an order of magnitude. Therefore, besides the particle volumetric concentration α, considered in that proposal, there must be other factors which also play an important role in the establishment of the drag coefficient for particles in a turbulent two-phase suspension flow.

Lee (1987a) has analyzed the aforementioned results of local measurements of upward turbulent flow in the solid particle–air two-phase suspension of Lee & Durst (1982) and Tsuji *et al.* (1984) and obtained the realistic particle drag coefficient over a wide range of flow conditions. Lee (1987a) established that the particle drag in a turbulent suspension flow can be described by a pseudo Stokes law based on a turbulent particle Reynolds number for an extended range of values of this turbulent particle Reynolds number:

$$\mathrm{Re}_p = \frac{U_r d_p}{\tilde{\nu}_f}, \qquad [1]$$

where

$U_r = \bar{u}_f - \bar{u}_p$, the fluid-to-particle relative velocity,

$\bar{u}_f, \bar{u}_p$ = time-mean local longitudinal velocity component of the fluid and particles,

d_p = particle diameter

and

$\tilde{\nu}_f$ = apparent turbulent kinematic viscosity of the fluid felt by a particle in the suspension flow.

A correlation of the following functional form has been presented for the apparent turbulent kinematic viscosity of the fluid felt by a particle in the suspension flow $\tilde{\nu}_f$:

$$\frac{\tilde{\nu}_f}{\nu_f} = 500\alpha^{0.5}\mathrm{Fr}^{-2.33}(\mathrm{Re}')S^{0.3}, \quad \text{for} \quad 10 < \mathrm{Re}_p \leqslant 2 \times 10^3$$

$$\frac{\tilde{\nu}_f}{\nu_f} = \frac{u'_f}{(u'_f)_c}, \quad \text{for} \quad \mathrm{Re}_p \leqslant 10, \qquad [2]$$

where

$\mathrm{Fr} = \dfrac{U_0}{(d_p g)^{\frac{1}{2}}}$, the particle Froude number,

g = gravitational acceleration

U_0 = time-mean longitudinal velocity of the fluid along the pipe axis,

$\mathrm{Re}' = \dfrac{D u'_f}{\nu_f}$, the local flow turbulence Reynolds number,

D = pipe diameter,
u'_f = fluctuation of longitudinal fluid velocity,
$(u'_f)_c$ = value of u'_f in the central core,
ν_f = molecular kinematic viscosity of the fluid,

$S = \frac{\rho_p}{\rho_f}$, ratio of intrinsic densities of particles and fluid,
ρ_p = intrinsic density of particles

and

ρ_f = intrinsic density of the fluid.

TRANSVERSE DYNAMIC INTERACTION BETWEEN PHASES (PARTICLE MIGRATION)

An insight into the flow structure of the fluid in a turbulent dilute two-phase suspension flow can be obtained, for example, from the formulation of a new unified theory on the transverse turbulent transport of particles by Lee & Wiesler (1987). According to this theory, the equation governing the transverse transport of particles contains for the various force terms the following listed parameters which are expected to be available from the flow structure of the fluid in the two-phase suspension flow:

drag force term: $\tilde{a}, \bar{v}_f$;

lift force term: $\tilde{h}, \bar{u}_f, \frac{\partial \bar{u}_f}{\partial r}$;

turbulent diffusion force term: $\tilde{a}, \omega_e, \tilde{\eta}_e, v'_f$;

where

x, r = longitudinal and transverse coordinates, respectively,
$\bar{u}_f, \bar{v}_f$ = local time-mean components of fluid velocity in the x- and r-directions, respectively,
u'_f, v'_f = local fluctuations of fluid velocity in the x- and r-directions, respectively,
$\tilde{\eta}(\omega) = [(1+\tilde{f}_1)^2 + \tilde{f}_2^2]^{\frac{1}{2}}$, the ratio of the amplitude of oscillation of particle to that of the fluid in a turbulent suspension flow,

$$\tilde{f}_1(\omega) = \frac{\omega\left[\omega + \tilde{c}\left(\frac{\pi\omega}{2}\right)^{\frac{1}{2}}\right](b-1)}{\left[\tilde{a} + \tilde{c}\left(\frac{\pi\omega}{2}\right)^{\frac{1}{2}}\right]^2 + \left[\omega + \tilde{c}\left(\frac{\pi\omega}{2}\right)^{\frac{1}{2}}\right]^2},$$

$$\tilde{f}_2(\omega) = \frac{\omega\left[\tilde{a} + \tilde{c}\left(\frac{\pi\omega}{2}\right)^{\frac{1}{2}}\right](b-1)}{\left[\tilde{a} + \tilde{c}\left(\frac{\pi\omega}{2}\right)^{\frac{1}{2}}\right]^2 + \left[\omega + \tilde{c}\left(\frac{\pi\omega}{2}\right)^{\frac{1}{2}}\right]^2},$$

$$\tilde{a} = \frac{36\tilde{\nu}_f\rho_f}{(2\rho_p + \rho_f)d_p^2},$$

$$b = \frac{3\rho_f}{(2\rho_p + \rho_f)},$$

$$\tilde{c} = \frac{18\rho_f}{(2\rho_p + \rho_f)d_p}\left(\frac{\tilde{\nu}_f}{\pi}\right)^{\frac{1}{2}},$$

$$\tilde{h} = 1.615\tilde{\nu}_f^{\frac{1}{2}}d_p^2\frac{\rho_f}{(2\rho_p + \rho_f)},$$

ω = frequency of fluid oscillation,
ω_e = most energetic frequency of fluid oscillation

and

$\tilde{\eta}_e = \tilde{\eta}(\omega_e)$.

Inspection of these parameters reveals that in order to calculate the transverse transport of particles in a turbulent two-phase suspension flow, inputs on the quantities u_f', v_f', ω_e. $\bar{u}_f$, $\partial \bar{u}_f/\partial r$, $\bar{v}_f$ and $\tilde{v}_f$ are needed from the flow structure of the fluid phase in such a flow. The apparent turbulent kinematic viscosity of the fluid felt by a particle in a turbulent two-phase suspension flow, $\tilde{v}_f$, has been formulated from the correlation of Lee (1987a) in terms of physical and dynamic properties. For simple flow in a straight pipe, the time-mean transverse fluid velocity, $\bar{v}_f$, is identically zero. The remaining quantities, u_f', v_f', ω_e, $\bar{u}_f$ and $\partial \bar{u}_f/\partial r$, are obtained from the experimental results of Lee & Durst (1982) and Tsuji *et al.* (1984).

ANALYSIS OF THE FLUID FLOW STRUCTURE IN A TWO-PHASE SUSPENSION FLOW

Longitudinal Fluctuation Velocity of the Fluid (u_f')

(*a*) *Shape of the transverse profile:*

$$\frac{u_f'}{U_0} = f\left(\frac{r}{R}\right).$$

The longitudinal fluctuation velocity of the fluid, u_f', has been found to vary distinctively in two transverse regions: the central core and the near-wall region. In the central core, u_f' remains essentially invariant. In the near wall region, u_f' first rises to a maximum and then falls to zero at the wall. The following simplified model, as shown in figure 1, is proposed:

for

$$0 \leqslant \frac{r}{R} \leqslant \frac{r_c}{R}, \quad \frac{u_f'}{U_0} = \frac{(u_f')_c}{U_0}; \qquad [3]$$

for

$$\frac{r_c}{R} \leqslant \frac{r}{R} \leqslant \frac{r_{max}}{R}, \quad \frac{u_f'}{U_0} = \frac{(u_f')_c}{U_0} + \left[\frac{(u_f')_{max}}{U_0} - \frac{(u_f')_c}{U_0}\right] \frac{\left(\frac{r}{R} - \frac{r_c}{R}\right)}{\left(\frac{r_{max}}{R} - \frac{r_c}{R}\right)}; \qquad [4]$$

for

$$\frac{r_{max}}{R} \leqslant \frac{r}{R} \leqslant 1, \quad \frac{u_f'}{U_0} = \left[\frac{(u_f')_{max}}{U_0}\right] \frac{\left(1 - \frac{r}{R}\right)}{\left(1 - \frac{r_{max}}{R}\right)}; \qquad [5]$$

where

$(u_f')_c$ = value of u_f' in the central core,
$(u_f')_{max}$ = maximum value of u_f' in the near-wall region,
r_c = transverse location where the central core joins the near-wall region,
r_{max} = transverse location in the near-wall region where u_f' attains the maximum value $(u_f')_{max}$,
R = pipe radius

and

U_0 = time-mean longitudinal velocity of the fluid along the axis.

(*b*) *Size of the central core:*

$$\left(\frac{r_c}{R}\right),$$

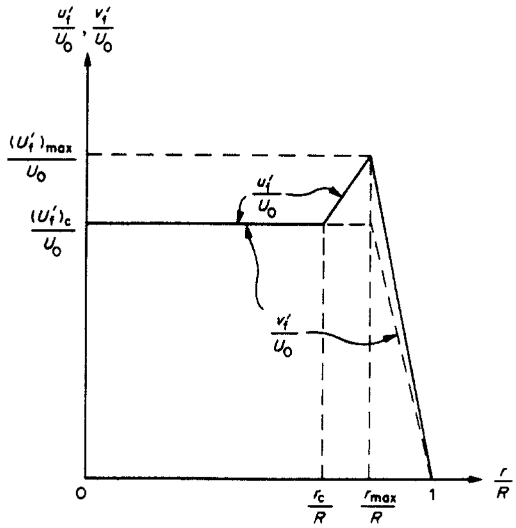

Figure 1. Sketch of the proposed transverse profiles of the fluctuation velocities of the fluid, u_f' and v_f' in a dilute turbulent two-phase suspension flow in a vertical pipe.

The following correlation has been found for the size of the central core:

$$\frac{r_c}{R} = \left[\left(\frac{S}{833}\right)^{1.4} \exp(-1600\mathrm{Fr}^{-2}) - 0.74\right]\left(\frac{\mathrm{Re}}{2.2 \times 10^4}\right)^{2.5} m + 0.05, \qquad [6]$$

where m = ratio of mass fluxes of the particles and fluid. A comparison of this correlation, [6], with the experimental results is shown in figure 2.

(*c*) *Inside the central core:* $(u_f')_c$

The following correlation has been found for the longitudinal fluctuation velocity of the fluid in the central core:

$$\frac{(u_f')_c}{U_0} = [(\mathrm{Fr}^{-1} - 0.0034)^{0.75} - 0.0075]m\left(\frac{2.2 \times 10^4}{\mathrm{Re}}\right)^{0.77}\left(\frac{833}{S}\right) + 0.05\left(\frac{2.2 \times 10^4}{\mathrm{Re}}\right)^{0.14}, \qquad [7]$$

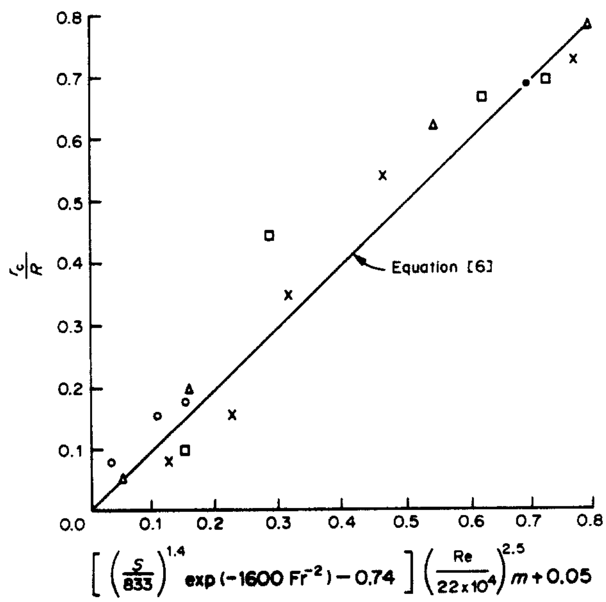

Figure 2. Comparison between the correlation and the experiments on the size of the central core for the longitudinal fluctuation velocity of the fluid, u_f'. Tsuji *et al.* (1984): ○, $d_p = 3$ mm, Fr = 81.3, $S = 833$, Re = 2.2 = 10^4; △, $d_p = 1$ mm, Fr = 133.3, $S = 833$, Re = 2.6×10^4; □, $d_p = 0.5$ mm, Fr = 185.2, $S = 833$, Re = 2.2×510^4; ×, $d_p = 0.2$ mm, Fr = 294.1, $S = 833$, Re = 2.3×10^4. Lee & Durst (1982): ●, $d_p = 0.8$ mm, Fr = 63.9, $S = 1833$, Re = 1.21×10^4.

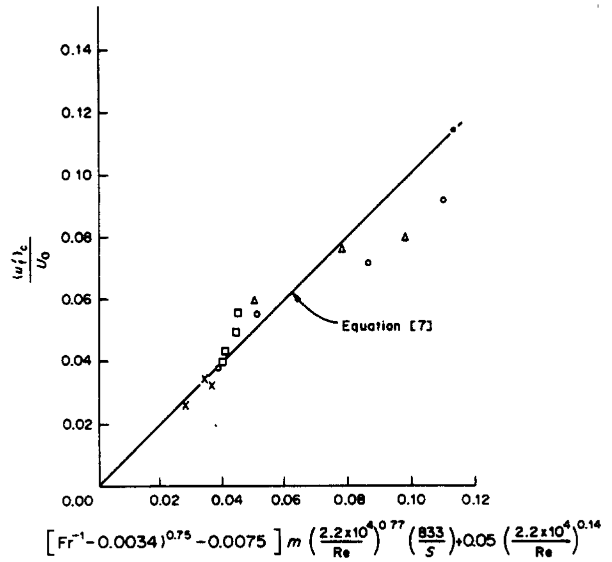

Figure 3. Comparison between the correlation and the experiments on the longitudinal fluctuation velocity of the fluid inside the central core $(u_f')_c$. Key as in figure 2.

where $(u_f')_c$ = value of u_f' in the central core. A comparison of this correlation with the experimental results is shown in figure 3.

(*d*) *Location of maximum fluctuation:*

$$\left(\frac{r_{max}}{R}\right).$$

Within the range of the physical and flow parameters covered by these experiments, the normalized transverse location of the maximum u_f' in the near-wall region, r_{max}/R, seems to be constant and the following correlation is formulated:

$$\frac{r_{max}}{R} \doteq 0.933. \qquad [8]$$

(*e*) *Maximum fluctuation:* $(u_f')_{max}$

The following correlations have been found for the maximum value of u_f' in the near-wall region $(u_f')_{max}$:

$$\frac{(u_f')_{max}}{U_0} = \frac{(u_f')_{max,0}}{U_0}\left\{1 + \left[3.23\,Fr^{0.345}\left(\frac{2.2\times10^4}{Re}\right)^{0.36}\left(\frac{S}{833}\right)^{0.25} - 0.6\right]m\left(\frac{833}{S}\right)^{2.4}\right\} \qquad [9]$$

and

$$\frac{(u_f')_{max,0}}{U_0} = 0.1\left(\frac{8.913\times10^4}{Re}\right)^{0.0588}, \qquad [10]$$

where $(u_f')_{max,0}$ = value of $(u_f')_{max}$ for the corresponding single-phase fluid flow. Comparisons of these correlations, [9] and [10], with the experimental results are shown in figures 4 and 5.

Transverse Fluctuation Velocity of the Fluid (v_f')

Since there has been no measurement of v_f' in the two experiments under consideration, assumptions have to be made using the results of measurements on the corresponding single-phase fluid flow. From the single-phase fluid flow measurement of Laufer (1953), v_f' is found to be approximately the same as u_f' for much of the central core. In the near-wall region, while u_f' first raises to a maximum and then falls to zero at the wall, v_f' falls steadily to zero at the wall. The

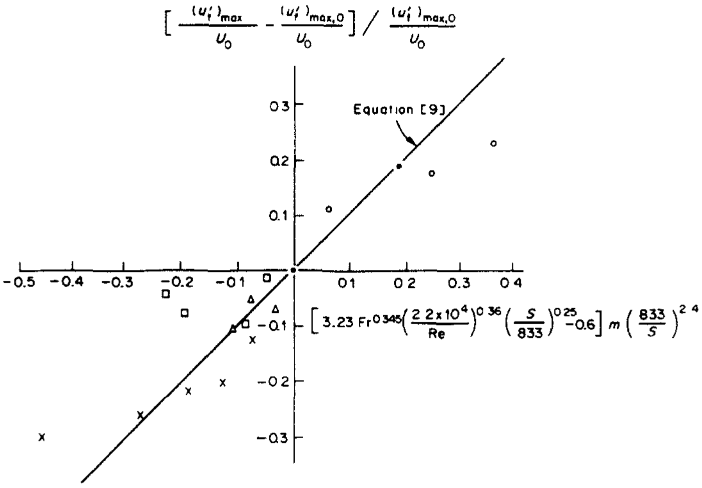

Figure 4. Comparison between the correlation and the experiments on the maximum longitudinal fluctuation velocity of the fluid in the near-wall region $(u_f')_{max}$. Key as in figure 2.

following transverse profile, for v_f' is thus proposed for the fluid in a turbulent two-phase suspension flow:

for

$$0 \leqslant \frac{r}{R} \leqslant \frac{r_{max}}{R}, \quad \frac{v_f'}{U_0} \doteq \frac{(v_f')_c}{U_0} \doteq \frac{(u_f')_c}{U_0}; \qquad [11]$$

and

for

$$\frac{r_{max}}{R} \leqslant \frac{r}{R} \leqslant 1, \quad \frac{v_f'}{U_0} = \frac{(u_f')_c}{U_0} \frac{\left(1 - \frac{r}{R}\right)}{\left(1 - \frac{r_{max}}{R}\right)}. \qquad [12]$$

Most Energetic Eddy Oscillation Frequency (ω_e)

Tsuji *et al.* (1984) measured the frequency power spectrum of air turbulence at three radial locations ($r/R = 0, 0.521$ and 0.912) for a total of four particle sizes ($d_p = 0.2$, 0.5, 1 and 3 mm)

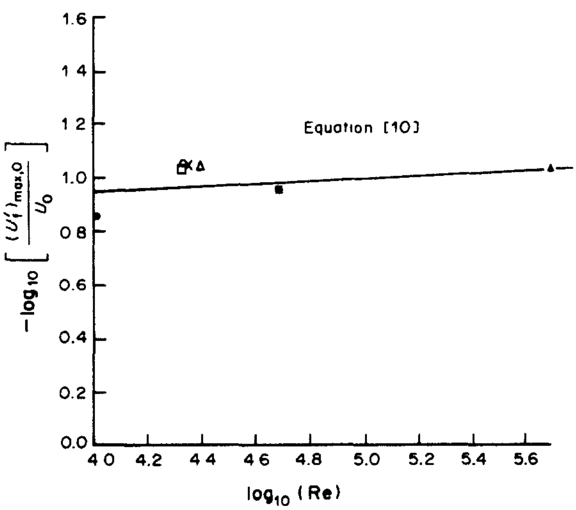

Figure 5. Comparison between the correlation and the experiments on the maximum longitudinal fluctuation velocity of the fluid in the near-wall region for single-phase flow $(u_f')_{max,0}$. Laufer (1953): ■, $Re = 5 \times 10^5$, ▲, $Re = 5 \times 10^5$. Other symbols as in figure 2.

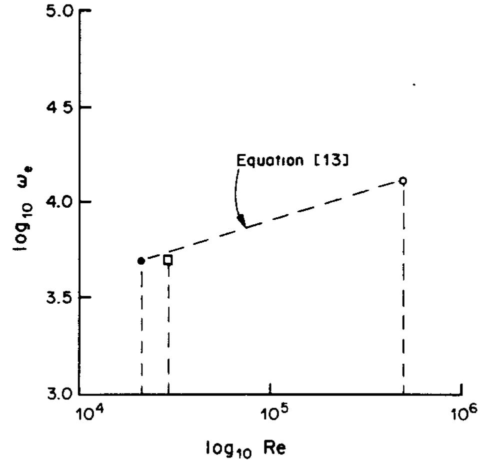

Figure 6. Comparison between the correlation and the results deduced from the measured frequency power spectrum of the longitudinal velocity of the fluid on the most energetic eddy oscillation frequency ω_e. Laufer (1953): ○, circular pipe; □, 2-D channel. Tsuji *et al.* (1984): ●, circular pipe.

and for each particle size with three values of the ratio of particle-to-air mass fluxes. When the measured power spectrum is replaced by a uniform power distribution of air turbulence over the same frequency range, the most energetic eddy oscillation frequency ω_e can be readily obtained. This is done by replacing the measured turbulence power spectrum with a flat-top turbulence power spectrum which produces an identical area of integration. The intersection of these two spectrum curves gives ω_e. Results of this most energetic eddy oscillation frequency are shown in table 1. The

Table 1. Most energetic eddy oscillation frequency ω_e deduced from the measured frequency power spectrum of the longitudinal velocity of the fluid in turbulent single-phase and dilute two-phase suspension flows in a vertical pipe

(a) From Tsuji *et al.*'s (1984) data

Flow Reynolds number, Re	Mass flux ratio, m	Particle diameter, d_p (mm)	Transverse location $\frac{r}{R}=0$	$\frac{r}{R}=0.521$	$\frac{r}{R}=0.912$
2.3×10^4 (Pipe)	0	(Single phase)	$\log_{10}\omega_e = 3.69$[a]	3.62	3.58
	1.3	0.2	3.64	3.61	3.62
	3.2	0.2	3.81	3.79	3.66
2.2×10^4 (Pipe)	0	(Single phase)	3.71	3.64	3.61
	1.3	0.5	3.77	3.68	3.65
	3.4	0.5	3.72	3.77	3.70
2.3×10^4 (Pipe)	0	(Single phase)	3.65	3.60	3.58
	0.6	1.0	3.64	3.60	3.63
	3.0	1.0	3.65	3.63	3.65
2.2×10^4 (Pipe)	0	(Single phase)	3.68	3.64	3.59
	1.1	3.0	3.60	3.58	3.59
	3.4	3.0	3.58	3.60	3.60

(b) From Laufer's (1953) data

Flow Reynolds number, Re	Transverse location $\frac{r}{R}=0$ (centerline)
5×10^4 (Single phase) (Pipe)	$\log_{10}\omega_e = 4.28$
3.08×10^4 (Single phase) (2-D channel)	3.70

[a]In rad/s.

flow Reynolds number for these experiments remains fairly constant at around a mean value of $\mathrm{Re} = 2.25 \times 10^4$. It is evident that the most energetic eddy oscillation frequency ω_e stays practically constant at around the same value, about 5×10^3 (or $\log_{10} \omega_e = 3.70$), as for single-phase flow for all particle sizes and loadings at all radial locations.

Laufer (1953) used hot-wire anemometry to measure the frequency power spectrum of air turbulence in a single-phase pipe flow at a flow Reynolds number of $\mathrm{Re} = 5 \times 10^5$. The most energetic eddy oscillation frequency obtained in a similar fashion is $\omega_e \doteq 1.9 \times 10^4$ (or $\log_{10} \omega_e \doteq 4.28$). For a two-dimensional single-phase flow at a flow Reynolds number of $\mathrm{Re} = 3.08 \times 10^4$, the most energetic eddy oscillation frequency is $\omega_e \doteq 5 \times 10^3$ (or $\log_{10} \omega_e \doteq 3.70$). A plot of the most energetic eddy oscillation frequency vs Reynolds number, which is assumed to be valid for single-phase flows as well as two-phase flows, is shown in figure 6. The following correlation is formulated:

$$\omega_e = 67.6\,\mathrm{Re}^{0.43}. \qquad [13]$$

Time-mean Longitudinal Velocity of the Fluid (u_f)

Lee & Durst (1982) measured the time-mean air velocity of a single-phase air flow as well as a glass particle–air two-phase flow with four different particle sizes in a vertical pipe. Tsuji *et al.*, (1984) measured the time-mean air velocity of a single-phase air flow with three different particle sizes with three different particle loadings, each in a vertical pipe. The following correlations have been formulated for the time-mean longitudinal velocity of the fluid in a turbulent two-phase suspension flow in a vertical pipe $\bar{u}_f$:

(*a*) *along the pipe axis*

$$\frac{(\bar{u}_f)_c}{(\bar{u}_f)_{c,0}} = 1 - \left[0.076 - 1086\,\mathrm{Fr}^{-2}\left(\frac{833}{S}\right)^{1.88}\right] m; \qquad [14]$$

(*b*) *shape of the transverse profile*

$$\frac{\bar{u}_f}{(\bar{u}_f)_c} = \exp\left[3\left(\frac{r}{R}\right)^{0.51}\right]\left[1 - \left(\frac{r}{R}\right)^{0.0133m\left(\frac{833}{S}\right)^{0.8} + 0.17}\right]^{\left[1 + 2.4\left(\frac{r}{R}\right)^2\left(1 - \frac{r}{R}\right)^2\right] / \left[1 + 0.25\left(\frac{r}{R}\right)\right]}; \qquad [15]$$

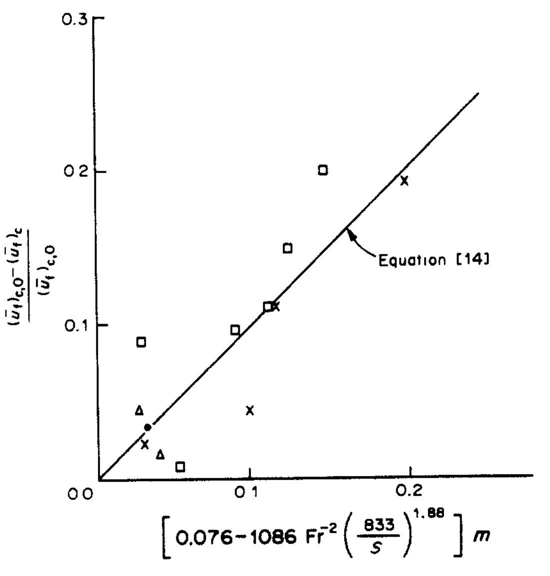

Figure 7. Comparison between the correlation and the experiments on the time-mean longitudinal velocity of the fluid along the pipe axis $(\bar{u}_f)_c$. Tsuji *et al.* (1984): △, $d_p = 1$ mm, $\mathrm{Fr} = 81.3$, $S = 833$, $\mathrm{Re} = 2.2 \times 10^4$; □, $d_p = 0.5$ mm, $\mathrm{Fr} = 133.3$, $S = 833$, $\mathrm{Re} = 2.6 \times 10^4$; ×, $d_p = 0.2$ mm, $\mathrm{Fr} = 294.1$, $S = 833$, $\mathrm{Re} = 2.3 \times 10^4$; Lee & Durst (1982): ●, $d_p = 0.8$ mm, $\mathrm{Fr} = 63.9$, $S = 1833$, $\mathrm{Re} = 1.21 \times 10^4$.

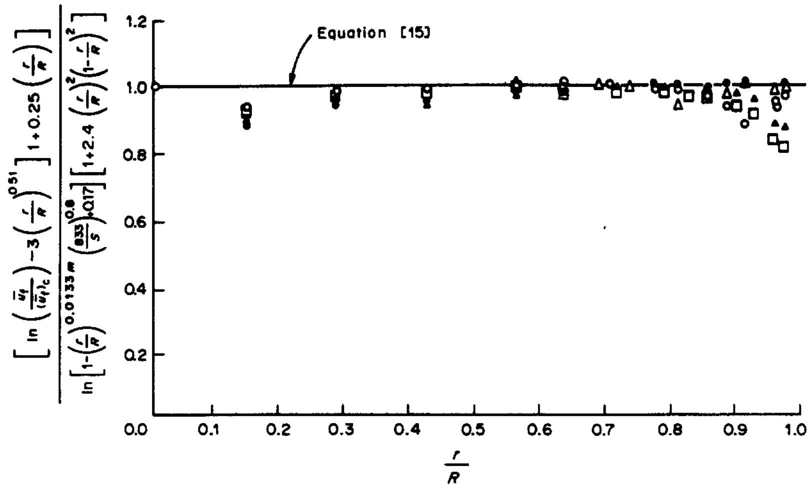

Figure 8. Comparison between the correlation and the experiments on the transverse distributions of the time-mean longitudinal velocity of the fluid, $\bar{u}_f$. Lee & Durst (1982): $S = 1833$, $Re = 1.21 \times 10^4$; ○, $d_p = 0.8$ mm, $m = 1.21$; △, $d_p = 0.4$ mm, $m = 1.72$; ▲, $d_p = 0.2$ mm, $m = 0.63$; □, $d_p = 0.1$ mm, $m = 0.58$; ●, (single phase), $m = 0$.

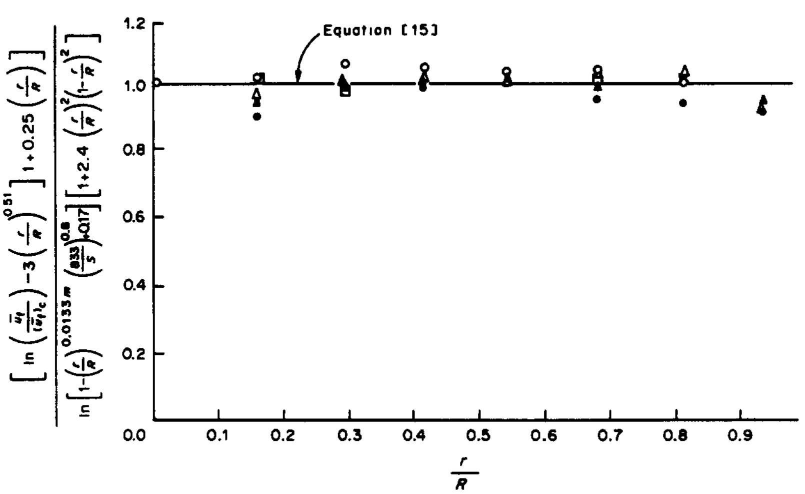

Figure 9. Comparison between the correlation and the experiments on the transverse distribution of the time-mean longitudinal velocity of the fluid, $\bar{u}_f$. Tsuji *et al.* (1984): $d_p = 0.2$ mm, $S = 833$, $Re = 2.3 \times 10^4$; ○, $m = 3.2$; △, $m = 1.9$; ▲, $m = 1.3$; □, $m = 0.5$; ●, $m = 0$ (single phase).

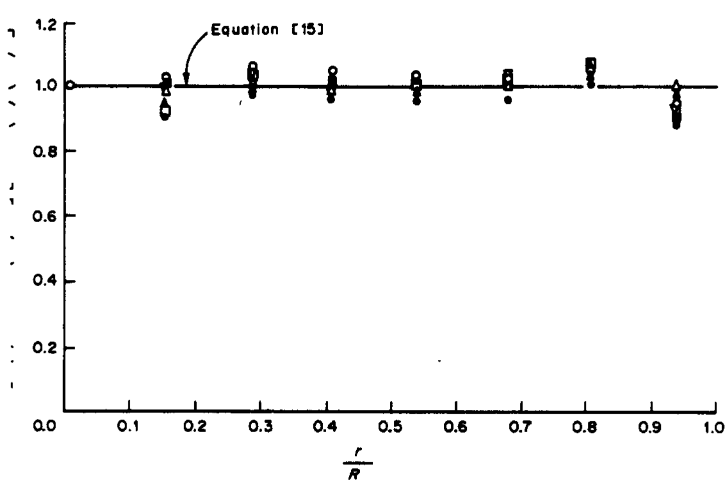

Figure 10. Comparison between the correlation and the experiments on the transverse distribution of the time-mean longitudinal velocity of the fluid, $\bar{u}_f$. Tsuji *et al.* (1984): $d_p = 0.5$ mm, $S = 833$, $Re = 2.2 \times 10^4$; ○, $m = 3.4$; ■, $m = 2.9$; ▽, $m = 2.6$; △, $m = 2.1$; ▲, $m = 1.3$; □, $m = 0.7$; ●, $m = 0$ (single phase).

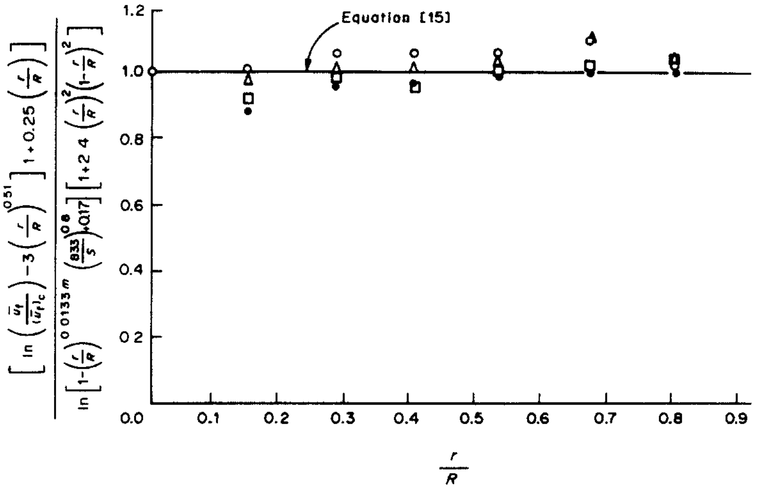

Figure 11. Comparison between the correlation and the experiments on the transverse distribution of the time-mean longitudinal velocity of the fluid $\bar{u}_f$. Tsuji *et al.* (1984): $d_p = 1$ mm, $S = 833$, $Re = 2.6 \times 10^4$; ○, $m = 3.0$; △, $m = 2.0$; □, $m = 0.6$; ●, $m = 0$ (single phase).

where

$\bar{u}_f$ = local time-mean longitudinal velocity of the fluid in a turbulent two-phase suspension flow in a vertical pipe,

$(\bar{u}_f)_c$ = value of $\bar{u}_f$ along the pipe axis

and

$(\bar{u}_f)_{c,0}$ = value of $\bar{u}_f$ along the pipe axis for the corresponding single-phase flow.

A comparison of the centerline value $(\bar{u}_f)_c$ between the correlation [14] and the experimental results is shown in figure 7. A comparison of the shape of the transverse profile of $\bar{u}_f$ between the correlation [15] and the experimental results is shown in figures 8–11.

CONCLUSIONS

For a dilute turbulent two-phase suspension flow in a vertical pipe, the following conclusions can be drawn:

1. Theoretical analysis requires, for such a flow system, the determination of the local apparent turbulent kinematic viscosity of the fluid felt by the particles, the local longitudinal and transverse fluctuation velocities of the fluid, the local most energetic eddy oscillation frequency of the fluid and the distribution of the local time-mean longitudinal velocity of the fluid.
2. The local apparent turbulent kinematic viscosity of the fluid felt by the particles has been previously determined in terms of the physical and flow properties of such a flow system from an analysis of the apparent drag between the phases found in local measurements.
3. The remaining flow properties of the fluid in such a flow system have been determined from an analysis of the results of some recent local measurements. Correlations for the local longitudinal and transverse fluctuation velocities of the fluid, the local most energetic eddy oscillation frequency of the fluid and the distribution of the local time-mean longitudinal velocity of the fluid have been found in terms of the physical and flow properties of such a flow system. Comparisons of these correlations with the experiments are provided.
4. The local particle volumetric concentration α is expected to play an important role in the flow structure of the carrier fluid in the two-phase suspension. It has not been explicitly used due to a lack of direct measurements from available

experiments. The quantity α appears in the turbulent particle diffusion force term of Lee (1987a) when applied to a pipe flow:

$$-\frac{2\pi}{\omega_e}\tilde{\alpha}\tilde{\eta}_e^2(v_f')^2\frac{1}{\alpha}\frac{\partial\alpha}{\partial r}. \qquad [16]$$

With Lee's (1987a) theory on particle transport, using the apparent turbulent viscosity of the fluid provided by the correlation of Lee (1987b, this issue, pp. 247–256), and the turbulent flow properties of air in a two-phase suspension flow, provided by the correlations of the present work, Lee & Wong (1987) have performed numerical calculations of particle motion in an upward turbulent glass particle–air two-phase suspension flow in a vertical pipe. One of their results is the distribution of the local particle volumetric concentration α. Other results are the distributions of the air and particle mean velocities. These calculated velocities are found to agree very closely with measurements of Lee & Durst (1982), particularly with the particle migratory behavior according to particle size in the near-wall region. This verification lends support to the implicit participation of suspended particles in the present correlations.

Acknowledgements—The authors would like to thank the Deutsche Forschungsgemeinschaft for its support and Professor F. Durst for his encouragement.

REFERENCES

ALEXANDER, L. G. & COLDREN, C. L. 1951 Droplet transfer from suspending air to duct wall. *Ind. Engng Chem.* **45,** 1325–1331.

BEAL, S. K. 1968 Transport of particles in turbulent flow to channel or pipe walls. Report No. WAPDTM-765, Bettis Atomic Power Lab., Westinghouse Electric Corp., Pittsburg, Pa.

COUSINS, L. B. & HEWITT, G. G. 1968 Liquid phase mass transfer in annular two-phase flow: droplet deposition and liquid entrainment. Report No. R-5657, UKAEA, Harwell, Oxon.

DAVIS, C. N. 1966 Deposition of aerosols through pipes. *Proc. R. Soc.* (*Lond.*) **A289,** 235–246.

FRIEDLANDER, S. K. & JOHNSTONE, H. F. 1957 Deposition of suspended particles from turbulent gas streams. *Ind. Engng Chem.* **49,** 1151–1156.

HJELMFELT, A. T. JR & MOKROS, L. F. 1966 Motion of discrete particles in a turbulent fluid. *Appl. scient. Res.* **16,** 149–161.

HUTCHINSON, P., HEWITT, G. F. & DUKLER, A. E. 1971 Deposition of liquid or solid despersion from turbulent gas streams: a stochastic model. Report No. AERE R 6637, UKAEA, Harwell, Oxon. (1970). Also *Chem. Engng Sci.* **26,** 419–439.

ISHII, M. 1977 One-dimensional drift-flux model and constitutive equations for relative motion between phases in various two-phase flow regimes. Report ANL-77-47, Argonne National Lab., Argonne, Ill.

ISHII, M. & ZUBER, N. 1979 Drag coefficient and relative velocity in bubbly, droplet or particulate flows. *AIChE Jl* **25,** 843–855.

LAUFER, J. 1953 The structure of turbulence in fully developed pipe flow. *NACA tech. Note 2954.*

LEE, S. L. 1987a A unified theory on particle transport in a turbulent dilute two-phase suspension flow—III. *Int. J. Multiphase Flow* **13**(1), 137–144.

LEE S. L. 1987b Particle drag in a dilute turbulent two-phase suspension flow. *Int. J. Multiphase Flow* **13**(2), 247–256.

LEE, S. L. & DURST, F. 1979 On the motion of particles in turbulent flows. Sonderforschungsbereich 80, Univ. of Karlsruhe, Karlsruhe, F.R.G.; SFB/80/TE/132.

LEE, S. L. & DURST, F. 1982 On the motion of particles in turbulent duct flows. *Int. J. Multiphase Flow* **8,** 125–146.

LEE, S. L. & WIESLER, M. A. 1987 Theory on transverse migration of particles in a turbulent two-phase suspension flow due to turbulent diffusion—I. *Int. J. Multiphase Flow* **13**(1), 99–111.

LEE, S. L. & WONG, C. L. 1987 A numerical calculation of transverse migration of particles in a dilute turbulent two-phase suspension flow in a vertical pipe. Submitted for publication.

LIN, C. S., MOULTON, R. W. & PUTNAM, G. L. 1953 Mass transfer between solid wall and fluid streams. *Ind. Engng Chem.* **45**, 667–678.

ROUHIAINEN, P. O. & STACHIEWICZ, J. W. 1970 On the deposition of small particles from turbulent streams. *J. Heat Transfer* **92**, 169–177.

TCHEN, C. M. 1949 Mean value and correlation problems connected with the motion of small particles suspended in a turbulent field. Ph.D. Thesis, Delft, The Netherlands.

TSUJI, Y., MORIKAWA, A. & SHIOMI, H. 1984 LDV measurements of an air–solid two-phase flow in a vertical pipe. *J. Fluid Mech.* **139**, 417–434.